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4-PRIME CORDIAL LABELING OF SOME DEGREE SPLITTING GRAPHS

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Abstract

Let G be a (p,q) graph. Let f : V(G) → {1,2,…,k} be a map. For each edge uv, assign the label gcd (f(u),f(v)). f is called k-prime cordial labelling of G if $|v_f(i)-v_f(j)| \leq 1$, $i,j \in \{1,2,\ldots,k\}$ and $|ef(0) - ef(1)| \leq 1$ where $v_f(x)$ denotes the number of vertices labelled with x,$e_f(1)$ and $e_f(0)$ respectively denote the number of edges labelled with 1 and not labelled with 1. A graph with a k-prime cordial labelling is called a k-prime cordial graph. In this paper we investigate 4-prime cordial labelling behaviour of degree splitting graph of path, jelly fish, crown and bistar and some more graphs.

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1. Introduction

In this paper graphs are finite, simple and undirected. Let G be a (p, q) graph where p refers the number of vertices of G and q refers the number of edge of G. The number of vertices of a graph G is called order of G, and the number of edges is called size of G. The concept of degree splitting graph was introduced by R. Ponraj and S.Somasundaram in [5]. Let G = (V,E) be a graph with $V = S_1 \cup S_2 \cup \cdots \cup S_t \cup T$ where each S_i is a set of vertices having at least two vertices and having the same degree and $T = V - \bigcup_{i=1}^{t} S_i$. The degree Splitting graph of G denoted by DS (G) is obtained from G by adding vertices $w_1, w_2 \ldots, w_t$ and joining w_i to each vertex of S_i $(1 \leq i \leq t)$. Let G_1, G_2 respectively be (p_1, q_1), (p_2, q_2) graphs. The corona of G_1 with G_2, $G_1 \odot G_2$ is the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2. The bistar $B_{m,n}$ is the graph obtained by making adjacent the two central vertices of $K_{1,m}$ and $K_{1,n}$. Jelly fish graphs J (m, n) obtained from a cycle C_4 : uvxyu by joining x and y with an edge and appending m pendent edges to u and n pendent edges to v. In 1987, Cahit introduced the concept of cordial labelling of graphs [1]. Sundaram, Ponraj, Somasundaram [6] have introduced the notion of prime cordial labeling. Also they discussed the prime cordial labeling behaviour of various graphs. Recently Ponraj et al. [8], introduced k-prime cordial labeling of graphs. In [9, 10] Ponraj et al. studied the 4-prime cordial labeling behaviour of complete graph, book, flower, mC_n, wheel, gear, double cone, helm, closed helm, butterfly graph, and friendship graph and some more graphs. In this paper we study about the 4-prime cordiality of degree splitting graph of path, jelly fish graph, crown, bistar, subdivision of a star, subdivision of bistar ad subdivision of crown. A binary vertex labeling $f : V(G) \rightarrow \{0, 1\}$ of graph G with induced edge labeling $f : E(G) \longrightarrow \{0, 1\}$ defined by $f(uv) = f(u)f(v)$ is called a product cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, where $v_f(0)$, $v_f(1)$ denote the number of vertices of G having labels 0, 1 respectively under f and $e_f(0)$, $e_f(1)$ denote the number of edges of G having labels 0, 1 respectively under f . A graph G is product cordial if it admits product cordial labeling [7]. Let x be any real number. Then $\lfloor x \rfloor$ stands for the largest integer less than or equal to x and $\lceil x \rceil$ stands for smallest integer greater than or equal to x. Terms that are not defined here, follow from Harary [3] and Gallian [2].

2. Main results

Observation 2.1. A 2-prime cordial labeling is a product cordial labeling. [7]

Proof: Obviously, since 2-Prime cordial labeling produces same vf(x) and ef(x) { x= 0,1} as in product cordial labeling.

Theorem 2.1. $DS(P_n)$ is 4-prime cordial for all n.

Proof. Let P_n be the path $u_1u_2 \dots u_n$. Let $V(DS(P_n)) = V(P_n) \cup \{u, v\}$ and $E(DS(P_n)) = E(P_n) \cup \{uu_1, uv_n, vu_i : 2 \le i \le n - 1\}$. Clearly $DS(P_n)$ has n + 2vertices and 2n − 1 edges. The proof is divided into four cases.

Case 1. $n \equiv 0 \pmod 4$.

Let n = 4t. Assign the label 2 to the vertices $u_1, u_2, \dots u_t$ and 4 to the vertices$u_{t+1}, u_{t+2}, \dots, u_{2t}$. Then assign the label 1 and 3 alternatively to the remaining vertices. Finally assign the label 2, 4 respectively to the vertices u, v.

Case 2. $n \equiv 1 \pmod 4$.

As in case 1 assign the label to the vertices u, v, $u_i (1 \le i \le n - 1)$. Finally assign the label 3 to the last vertex u_n.

Case 3. $n \equiv 2 \pmod 4$.

Assign the label to the vertices u, v, u_i, $(1 \le i \le n - 1)$ as in case 2. Then assign the label 1 to the vertex u_n. Finally interchange the labels of $u_{\frac{n}{2}+2}$ and $u_{\frac{n}{2}+3}$

Case 4. $n \equiv 3 \pmod 4$.

Assign the label to the vertices u, v, u_i, $(1 \le i \le n - 1)$ as in case 3. Then assign the label 2 to the vertex u_n.

Theorem 2.2. $DS(C_n \odot K_1)$ is 4-prime cordial for all values of n.

Proof. Let $V(C_n \odot K_1) = \{u_i, v_i : 1 \le i \le n\}$ and $E(Cn \odot K1) = \{u_iu_{i+1}, u_iv_i : 1 \le i \le n - 1\} \cup \{u_nu_1, u_nv_n\}$. The graph $DS(C_n \odot K_1)$ is obtained by adding the new vertices u, v and joining u to u_i $(1 \le i \le n)$, v to v_i $(1 \le i \le n)$. We nowgive the label to the vertices of $DS(C_n \odot K_1)$. Assign the label 2, 1 respectively tothe vertices u and v. Next assign the label 2 to the vertices $u_1, u_2, \dots, u_{\lfloor \frac{n}{2} \rfloor}$ and 4 to the vertices $u_{\lfloor \frac{n}{2} \rfloor+1}, u_{\lfloor \frac{n}{2} \rfloor+2}, \dots, u_n$. Next assign the label 1 to the vertices $v_1, v_2, \dots, v_{\lfloor \frac{n}{2} \rfloor}$ and 3 to the vertices $v_{\lfloor \frac{n}{2} \rfloor+1}, v_{\lfloor \frac{n}{2} \rfloor+2}, \ldots, v_n$. The table 1 given below establish the labelling f is a 4-prime cordial labelling.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$v_f(4)$	$e_f(0)$	$e_f(1)$
n is odd	$\frac{n+1}{2}$	$\frac{n+1}{2}$	$\frac{n+1}{2}$	$\frac{n+1}{2}$	2n	2n
n is even	$\frac{n}{2}$	$\frac{n+2}{2}$	$\frac{n}{2}$	$\frac{n+2}{2}$	2n	2n

Table 1.

Theorem 2.3. If $n \equiv 1, 3 \pmod 4$, then $DS(B_{n,n})$ is 4-prime cordial.

Proof. Let $V(DS(B_{n,n})) = \{u, v, x, y, u_i, v_i : 1 \leq i \leq n\}$ and $E(DS(B_{n,n})) = \{uv, uy, vy, uu_i, vv_i, xu_i, xv_i : 1 \leq i \leq n\}$. Clearly $DS(B_{n,n})$ has $2n + 4$ verticesand $4n + 3$ edges.

Case 1. $n \equiv 1 \pmod 4$.

Let $n = 4t + 1$. Assign the label 2 to the vertices $u_1, u_2, \ldots, u_{2t+2}$. Next assignthe label 4 to the vertices $u_{2t+3}, u_{2t+4}, \ldots, u_{4t+1}$, x, y and u. Next assign thelabel 1 to the vertices $v_1, v_2, \ldots, v_{2t}$. Finally assign the label 3 to the vertices$v_{2t+1}, v_{2t+2}, \ldots, v_{4t+1}$.

Case 2. $n \equiv 3 \pmod 4$.

As in case 1 assign the label to the vertices u, v, x, y, u_i, v_i $(1 \leq i \leq n-2)$. Finallyassign the labels 2, 4, 1, 3 respectively to the vertices $u_{n-1}, u_n, v_{n-1}, v_n$. Obviously this vertex labelling is a 4-prime cordial labelling of $DS(B_{n,n})$, $n \equiv 1, 3 \pmod 4$ follows from Table 2.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$v_f(4)$	$e_f(0)$	$e_f(1)$
n = 4t+1	2t+1	2t+2	2t+1	2t+2	8t+1	8t+2
n = 4t+3	2t+2	2t+3	2t+2	2t+3	8t+3	8t+4

Table 2.

Theorem 2.4. Degree splitting graph of a subdivision of a star $K_{1,n}$, $DS(S(K_{1,n}))$ is 4-prime cordial for all values of n.

Proof. Let $V(DS(S(K_{1,n}))) = \{u, w_i, v_i, v, w : 1 \leq i \leq n\}$ and $E(DS(S(K_{1,n}))) = \{uw_i, w_iv_i, vv_i, ww_i : 1 \leq i \leq n\}$. Obviously $DS(K_{1,n})$ has $2n + 3$ vertices and 4nedges.

Case 1. n≡ 0 (mod 4).

Let n = 4t, t ∈ N. Consider the vertex u. Assign the label 2 to the vertex u. Next assign the labels 4, 1 to the vertices w, v respectively. We now consider the verticesof degree 3. Assign the label 2 to the vertices $w_1, w_2, \ldots, w_{2t}$ and 4 to the vertices$w_{2t+1}, w_{2t+2}, \ldots, w_{4t}$. Now we move to the vertices of degree 2. Assign the label 1to the vertices $v_1, v_2, \ldots, v_{2t}$ and 3 to the remaining vertices $v_{2t+1}, v_{2t+2}, \ldots, v_{4t}$.

Case 2. n≡ 1 (mod 4).

Let n = 4t + 1. As in case 1, assign the labels to the vertices u, v, w, v_i, w_i($1 \leq i \leq n - 1$). Finally assign the labels 4, 3 to the vertices w_n, v_n respectively.

Case 3. n≡ 2 (mod 4).

In this case assign the label to the vertices u, v, w, v_i, w_i ($1 \leq i \leq n-1$) as in case2. Next assign the labels 2, 1 to the vertices w_n, v_n respectively.

Case 4. n≡ 3 (mod 4).

As in case 3, assign the labels to the vertices u, v, w, v_i, w_i($1 \leq i \leq n-1$). Finally assign the labels 3, 4 to the remaining vertices v_n, w_n respectively.

The following table 3 establish that this vertex labeling f is a 4-prime cordial labeling.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$	$v_f(4)$	$e_f(0)$	$e_f(1)$
4t	2t+1	2t+1	2t	2t+1	8t	8t
4t+1	2t+1	2t+1	2t+1	2t+2	8t+1	8t+1
4t+2	2t+2	2t+2	2t+1	2t+2	8t+2	8t+2
4t+3	2t+2	2t+2	2t+2	2t+3	8t+3	8t+3

Table 3

Theorem 2.5. DS(J(n, n)) is 4-prime cordial.

Proof. Let V (DS(J(n, n))) = {u, v, x, y,w_1,w_2,w_3} ∪ {u_i, v_i : 1 ≤i≤ n}, and E(DS(J(n, n)) = {uy, vy, uw_1, uw_3, vw_1, vw_3,w_1w_2,w_2w_3} ∪ {uu_i, vv_i, xu_i, xv_i :1 ≤i≤ n}. Clearly DS(J(n, n)) has 2n + 7 vertices and 4n + 8 edges.

Case 1. n≡ 0 (mod 4).

Let n = 4t. Assign the label 2 to the vertices u_1, u_2, . . . , u_{2t+2}. Next assign thelabel 4 to the vertices u_{2t+3}, u_{2t+4}, . . . , u_4t. Then assign the label 4 to the verticesu, x, y, w_3. Assign the label 1 to the vertex v. Next assign the label 1 to thevertices v_1, v_2, . . . , v_{2t}. Finally assign the remaining non-labeled vertices with 3.

Case 2. n ≡1 (mod 4).

Let n = 4t + 1. In this case, assign the label 2 to the vertices u_1, u_2, . . . , u_{2t+3}. Next assign the label 4 to the vertices u_{2t+4}, u_{2t+5}, . . . , u_{4t+1}. Next assign the label 4 to the vertices x, u, v, y. Next assign the label 1 to the vertices v_1, v_2, . . . , v_{2t+2}. Finally assign the remaining non-labeled vertices with 3.

Case 3. n≡ 2 (mod 4).

As in case 2 assign the label to the vertices u_i, v_i (1 ≤i≤ n − 1), u, v, x, y, w_1, w_2, w_3. Finally assign the label 4, 3 respectively to the vertices u_n, v_n.

Case 4. n≡ 3 (mod 4).

Assign the label to the vertices u_i, v_i(1 ≤i≤ n − 1), u, v, x, y, w_1, w_2, w_3 as in case 3. Finally assign the label 2, 1 respectively to the vertices u_n, v_n.

The following table 4 establish that the above vertex labeling f is a 4-prime cordial labeling.

Nature of n	**$v_f(1)$**	**$v_f(2)$**	**$v_f(3)$**	**$v_f(4)$**	**$e_f(0)$**	**$e_f(1)$**
n ≡ 0 (mod 4)	2t+1	2t+3	2t+2	2t+2	8t+4	8t+4
n ≡1 (mod 4)	2t+2	2t+3	2t+2	2t+2	8t+6	8t+6
n ≡2 (mod 4)	2t+2	2t+3	2t+3	2t+3	8t+8	8t+8
n ≡3 (mod 4)	2t+3	2t+4	2t+3	2t+3	8t+10	8t+10

Table 4

Example 2.1. A 4-prime cordial labeling of J(5, 5) is given in Figure 1.

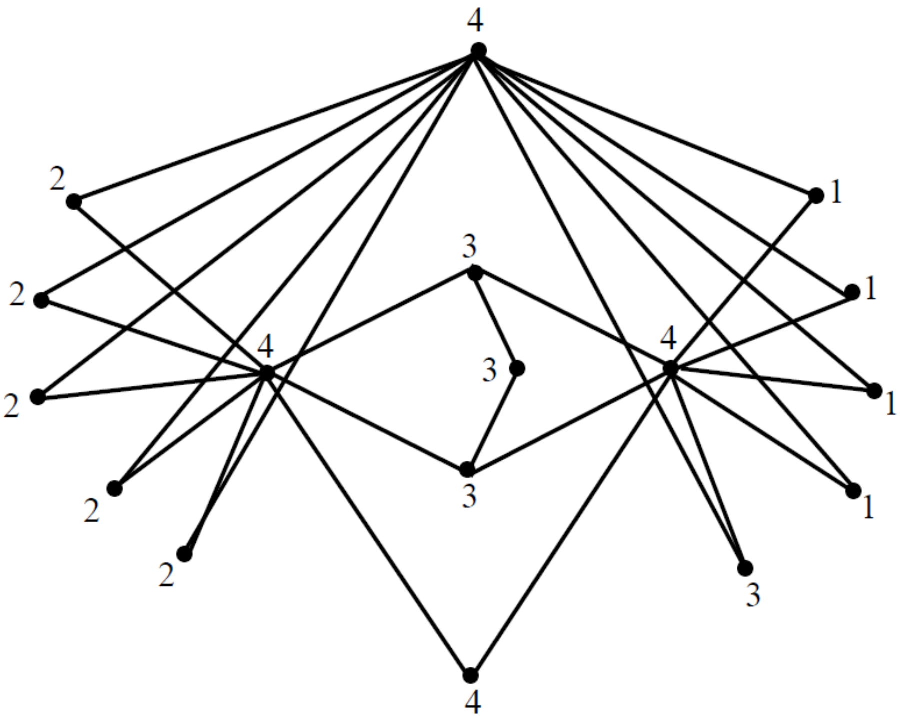

Figure 1

Example 2.2. A 4-prime cordial labeling of J(6, 6) is given in Figure 2.

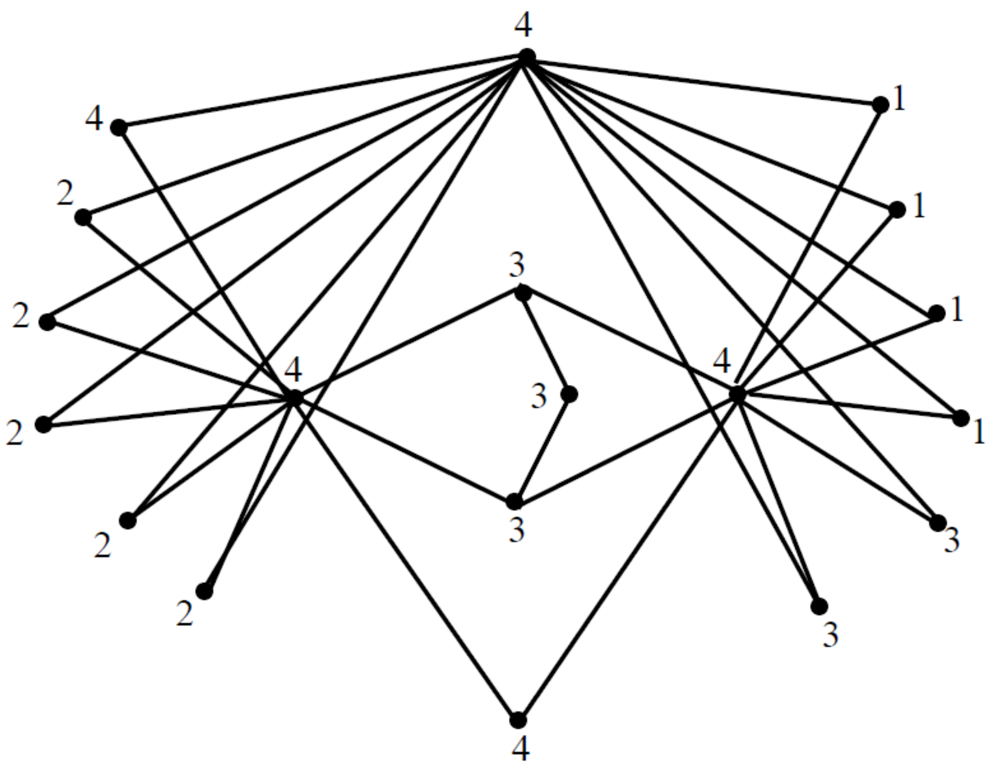

Figure 2

Example 2.3. A 4-prime cordial labeling of J(7, 7) is given in Figure 3.

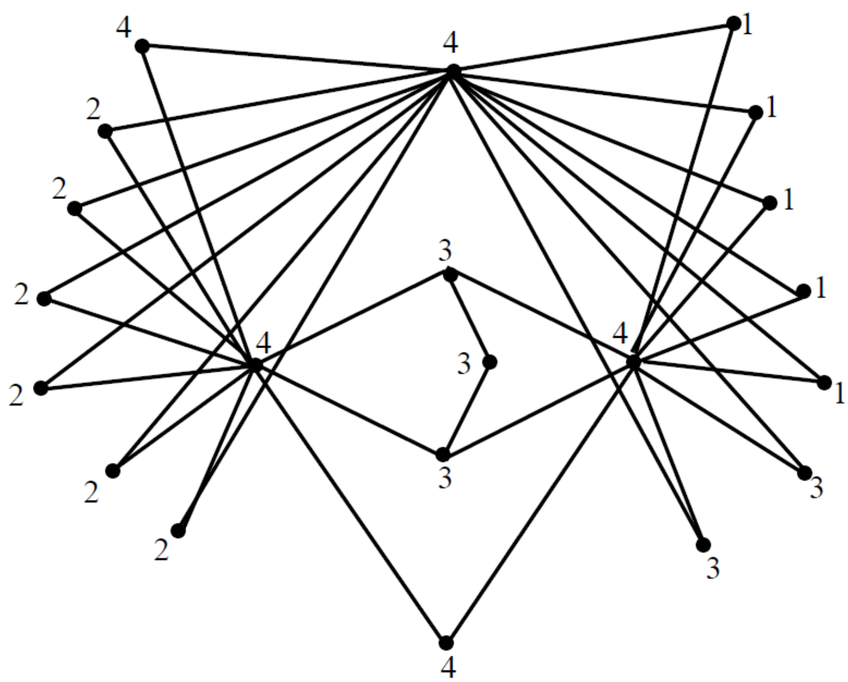

Figure 3

Theorem 2.6. DS(S($B_{n,n}$)) is 4-prime cordial for all values of n.

Proof. Let V (DS(S($B_{n,n}$))) = {u, v,w, x,w_1,w_2} ∪ {u_i, v_i, x_i, y_i: 1 ≤i≤ n} and E(DS(S($B_{n,n}$))) = {uu_i, u_ix_i, vv_i, v_iy_i,w_1x_i,w_1y_i,w_2u_i,w_2v_i : 1 ≤i≤ n}. It iseasy to verify that DS(S($B_{n,n}$)) has 4n + 6 vertices and 8n + 4 edges. Assign thelabel 2 to the vertices u, u_i (1 ≤i≤ n). Next consider the vertices x_i, assign the label 4 to the vertices xi (1 ≤i≤ n), w_1, w_2. Assign the label 3 to the verticesv$_1$, v_2, y_1, y_2, . . . , y_{n-1}. Finally assign the label 1 to all the non-labeled vertices. This vertex labeling f is a 4-prime cordial labeling since v_f (1) = v_f (4) = n + 2,v_f (2) = v_f (3) = n + 1 and e_f (0) = e_f (1) = 4n + 2.

Theorem 2.7. DS(S($C_n \odot K_1$)) is 4-prime cordial for all values of n.

Proof. We take the vertex set and edge set of $C_n \odot K_1$ as in Theorem 2.2. Let V (DS(S($C_n \odot K_1$))) = V ($C_n \odot K_1$) ∪ {w_1,w_2,w} ∪ {x_i, y_i : 1 ≤ i ≤ n} and E(DS(S($C_n \odot K_1$))) = {wx_i,wy_i, $y_iv_iu_ix_i$, u_iy_i,w_1v_i,w_2u_i : 1 ≤i≤ n}. Clearly DS(S($C_n \odot K_1$)) has 4n+3 vertices and 8n edges. We now give the 4-prime cordial labeling to this graph. Assign the label to the vertices u_i (1 ≤ i ≤ n) and 4 to the vertices x_i (1 ≤ i ≤ n). Assign the label 3 to the vertices v_i (1 ≤ i ≤ n) and to the vertex w_1. Finally assign the label 1 to the non-labeled vertices. If f is this vertex labeling then v_f (1) = v_f (2) = v_f (3) = n + 1, v_f(4) = n and e_f (0) = e_f (1) = 4n.This implies f is a 4-prime cordial labeling.

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